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A new large-update interior point algorithm for $P_*(\kappa)$ linear complementarity problems

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Abstract

In this paper we propose a new large-update primal-dual interior point algorithm for $P_*(\kappa)$ linear complementarity problems (LCPs). We generalize Bai et al.'s [A primal-dual interior-point method for linear optimization based on a new proximity function, Optim. Methods Software 17(2002) 985–1008] primal-dual interior point algorithm for linear optimization (LO) problem to $P_*(\kappa)$ LCPs. New search directions and proximity measures are proposed based on a kernel function which is not logarithmic barrier nor self-regular for $P_*(\kappa)$ LCPs. We showed that if a strictly feasible starting point is available, then the new large-update primal-dual interior point algorithm for solving $P_*(\kappa)$ LCPs has the polynomial complexity $O((1+2\kappa)n^{3/4}\log(n/\varepsilon))$ and gives a simple complexity analysis. This proximity function has not been used in the complexity analysis of interior point method (IPM) for $P_*(\kappa)$ LCPs before.

MSC: 49M15; 65K05; 90C33

Keywords: Large-update interior point method; Kernel function; Complexity; Polynomial algorithm; Linear complementarity problem

1. Introduction

In this paper we consider the following linear complementarity problem (LCP):

$$\begin{cases} s = Mx + q, \\ xs = 0, \\ x \geqslant 0, \quad s \geqslant 0, \end{cases} \tag{LCP}$$

where $M \in R^{n\times n}$ is a $P_*(\kappa)$ matrix and $q \in R^n$.

LCPs have many applications in mathematical programming and equilibrium problems. Indeed, it is known that by exploiting the first-order optimality conditions of the optimization problem, any differentiable convex quadratic program can be formulated into a monotone linear complementarity problem (LCP), i.e., $P_*(0)$ LCP, and vice versa [13]. And variational inequality problems are widely used in the study of equilibrium in, e.g., economics, transportation

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planning, and game theory. Variational inequality problems have a close connection to the LCPs. The reader can refer [5] for the basic theory, algorithms, and applications.

The primal-dual IPM for LO problem was first introduced in [7] and extended to various class of problems, e.g., [4,10]. Kojima et al. [7] first proved the polynomial computational complexity of the algorithm for LO problem, and since then many other algorithms have been developed based on the primal-dual strategy. Since IPMs follow the central path approximately, the existence of the central path is very important. Kojima et al. [8] proved the existence of the central path for any $P_*(\kappa)$ LCP and generalized the primal-dual interior point algorithm in [7] to $P_*(\kappa)$ LCP and they established the same complexity results. Since then a variant of an interior point algorithm's quality is measured by the fact whether it can be generalized to $P_*(\kappa)$ LCPs or not [6]. Miao [9] extended the Mizuno–Todd–Ye predictor–corrector method to $P_*(\kappa)$ LCPs. His algorithm uses the l_2-neighborhood of the central path and has $O((1+\kappa)\sqrt{n}L)$ iteration complexity. Recently, Illés and Nagy [6] give a version of the Mizuno–Todd–Ye predictor–corrector interior point algorithm for the $P_*(\kappa)$ LCP and show that the complexity of the algorithm is $O((1+\kappa)^{3/2}\sqrt{n}L)$. They choose τ and τ' neighborhood parameters in such a way that a predictor step following by one corrector step at each iteration. For larger value of κ the values of τ and τ' are fastly decreasing, therefore the constant in the complexity result is increasing.

Most of polynomial-time interior point algorithms for LO are based on the use of the logarithmic barrier function, e.g., [7,12]. Peng et al. [10] introduced self-regular barrier functions for primal-dual interior-point methods (IPMs) for LO and also extended to $P_*(\kappa)$ LCPs and proved the best complexity for large-update primal-dual IPMs for $P_*(\kappa)$ LCPs with some specific self regular barrier function. Recently Bai et al. [1,2] proposed new primal-dual IPMs for LO based on new proximity functions which are not logarithmic barrier and not self-regular.

In this paper we propose a new large-update primal-dual IPM which generalizes Bai et al.'s algorithm for LO to $P_*(\kappa)$ LCP and get the similar iteration complexity $O((1+2\kappa)n^{3/4}\log(n/\varepsilon))$ which is better than the classical large-update primal-dual algorithm based on the classical logarithmic barrier function. Since $P_*(\kappa)$ LCP is a generalization of LO problem, we lose the orthogonality of the vectors dx and ds. So our analysis is different from the one in [1]. The proximity measure plays an important role in the analysis of the algorithm and the proximity measure in this paper is first to use in the analysis of a large-update method for $P_*(\kappa)$ LCPs. And since we define a neighborhood and use a search direction based on the kernel function which is not logarithmic barrier and not self-regular, the analysis is different from the ones in [6,8–11]. Furthermore, our analysis provides a simpler way to analyze large-update IPMs.

This paper is organized as follows. In Section 2 we recall basic concepts and the notion of the central path. In Section 3 we describe the kernel function and its growth behavior. In Section 4 we compute the feasible step size and derive the amount of decrease of the proximity function during an inner iteration. In Section 5 we obtain the bound for the total number of iterations of the algorithm. Finally, concluding remarks are given in Section 6.

We use the following notations throughout the paper: R^n_+ denotes the set of n dimensional nonnegative vectors and R^n_{++}, the set of n dimensional positive vectors. For $x=(x_1,x_2,\ldots,x_n)^{\mathrm{T}}\in R^n$, $x_{\min}=\min\{x_1,x_2,\ldots,x_n\}$, i.e., the minimal component of x, $\|x\|$ is the 2-norm of x, and X is the diagonal matrix from vector x , i.e., $X=\mathrm{diag}(x)$. xs denotes the componentwise product (Hadamard product) of vectors x and s and $x^{\mathrm{T}}s$ is the scalar product of the vectors x and s. e is the n-dimensional vector of ones and I is the n-dimensional identity matrix. J is the index set, i.e., $J=\{1,2,\ldots,n\}$.

2. Preliminaries

$P_*(\kappa)$ matrix is introduced in [8] and we give definitions.

Definition 2.1. Let $\kappa\geqslant 0$ be a nonnegative number. A matrix $M\in R^{n\times n}$ is called a $P_*(\kappa)$ matrix if

$$(1+4\kappa)\sum_{i\in J_+(x)}x_i(Mx)_i+\sum_{i\in J_-(x)}x_i(Mx)_i\geqslant 0,$$

for all $x\in R^n$, where

$$J_+(x)=\{i\in J: x_i(Mx)_i\geqslant 0\}\quad\text{and}\quad J_-(x)=\{i\in J: x_i(Mx)_i<0\}.$$

Definition 2.2. A matrix $M \in R^{n\times n}$ is called a P_* matrix if it is a $P_*(\kappa)$ matrix for some $\kappa \geqslant 0$, i.e.,

$$P_* = \bigcup_{\kappa \geqslant 0} P_*(\kappa).$$

Note that the class P_* contains the class PSD of positive semi-definite matrices, i.e., matrices M satisfying $x^{\mathrm{T}}Mx \geqslant 0$ for all $x \in R^n$, and the class P of matrices with all the principal minors positive. In the following we give some definitions about convexity concepts which is essential in our analysis.

Definition 2.3 (*Boyd and Vanderberghe [3]*). A twice differentiable function $f : D(\subset R) \to R$ is strongly convex if and only if there exists $m_o > 0$ such that $f''(x) \geqslant m_o$ for all $x \in D$.

Definition 2.4 (*Peng et al. [10, Lemma 2.1.2]*). A function $f : D(\subset R) \to R$ is exponentially convex if and only if $f(\sqrt{x_1x_2}) \leqslant 1/2(f(x_1) + f(x_2))$ for all $x_1, x_2 \in D$.

We state some well-known results. For proofs and details see the book of Kojima et al. [8].

Proposition 2.5 (*Kajima et al. [8, Lemma 4.1]*). *If $M \in R^{n\times n}$ is a $P_*(\kappa)$ matrix, then*

$$M' = \begin{pmatrix} -M & I \\ S & X \end{pmatrix}$$

is a nonsingular matrix for any positive diagonal matrices $X, S \in R^{n\times n}$.

We use the following corollary to prove that the modified Newton system (NS) has a unique solution.

Corollary 2.6. *Let $M \in R^{n\times n}$ be a $P_*(\kappa)$ matrix and $x, s \in R^n_{++}$. Then for all $a \in R^n$ the system*

$$\begin{cases} -M\Delta x + \Delta s = 0, \\ S\Delta x + X\Delta s = a \end{cases}$$

has a unique solution $(\Delta x, \Delta s)$.

To find an approximate solution for (LCP) we relax the complementarity condition, i.e., the second equation in (LCP), and we get the following parameterized system:

$$\begin{cases} s = Mx + q, \\ xs = \mu e, \\ x > 0, \quad s > 0, \end{cases} \tag{CPP$_\mu$}$$

where $\mu > 0$. Without loss of generality, we assume that (LCP) is strictly feasible, i.e., there exists (x^0, s^0) such that $s^0 = Mx^0 + q, x^0 > 0, s^0 > 0$, and moreover, we have an initial strictly feasible point (x^0, s^0) such that $\Psi(x^0, s^0, \mu^0) \leqslant \tau$ for some $\mu^0 > 0$. For $P_*(\kappa)$ LCPs, it is not easy to find a strictly feasible point (x^0, s^0). In order to solve this difficulty, Kojima et al. [8] propose the big-M method to get a strictly feasible starting point for $P_*(\kappa)$ LCPs. For this given strictly feasible point (x^0, s^0) we can always find a $\mu^0 >$ such that $\Psi(x^0, s^0, \mu^0) \leqslant \tau$. Since M is a $P_*(\kappa)$ matrix and (LCP) is strictly feasible, (CPP$_\mu$) has a unique solution for any $\mu > 0$. We denote the solution of (CPP$_\mu$) as $(x(\mu), s(\mu))$ for given $\mu > 0$. We also call it μ-center for given μ and the solution set $\{(x(\mu), s(\mu)) \mid \mu > 0\}$ the central path of the (LCP). As $\mu \to 0$ the sequence $(x(\mu), s(\mu))$ approaches the solution(x, s) of the (LCP) [8]. We define the following notations:

$$d = \sqrt{\frac{x}{s}}, \quad v = \sqrt{\frac{xs}{\mu}}, \quad \mathrm{d}x = \frac{v\Delta x}{x}, \quad \mathrm{d}s = \frac{v\Delta s}{s}. \tag{2.1}$$

Then we have the scaled NS as follows:

$$\begin{cases} -\bar{M}\,\mathrm{d}x + \mathrm{d}s = 0, \\ \mathrm{d}x + \mathrm{d}s = v^{-1} - v, \end{cases} \tag{2.2}$$

where $\bar{M} = DMD$ and $D = \mathrm{diag}(d)$.

We consider a strictly convex function $\Psi(v)$ which is minimal at $v = e$ and $\Psi(e) = 0$. Then we replace the scaled centering equation, i.e., the second equation in (2.2), by

$$\mathrm{d}x + \mathrm{d}s = -\nabla\Psi(v). \tag{2.3}$$

So we get the following modified NS:

$$\begin{cases} -M\Delta x + \Delta s = 0, \\ S\Delta x + X\Delta s = -\mu v \nabla\Psi(v). \end{cases} \tag{NS}$$

This system uniquely defines a search direction $(\Delta x, \Delta s)$ by Corollary 2.6 since M is a $P_*(\kappa)$ matrix and (LCP) is strictly feasible by assumption. Throughout the paper we assume that a proximity parameter τ and a barrier update parameter θ are given and $\tau = \mathrm{O}(n)$ and $0 < \theta < 1$, fixed.

The algorithm works as follows. We assume that we are given a strictly feasible point (x, s) which is in a τ-neighborhood of the given μ-center. Then we decrease μ to $\mu_+ = (1 - \theta)\mu$, for some fixed $\theta \in (0, 1)$ and then we solve the modified NS to obtain the unique search direction. The positivity condition of a new iterate is ensured with the right choice of the step size α which is defined by some line search rule. This procedure is repeated until we find a new iterate (x_+, s_+) that is in a τ-neighborhood of the μ_+-center and then we let $\mu := \mu_+$ and $(x, s) := (x_+, s_+)$. Then μ is again reduced by the factor $1 - \theta$ and we solve the modified NS targeting at the new μ_+-center, and so on. This process is repeated until μ is small enough, say until $n\mu \leqslant \varepsilon$. Throughout the paper, we use the proximity function $\Psi(v)$ to find a search direction and to measure the proximity between the current iterates and the μ-center. Then we get the following algorithm.

Algorithm.

```
Input:
  A threshold parameter τ > 0;
  an accuracy parameter ε > 0;
  a fixed barrier update parameter θ, 0 < θ < 1;
  starting point (x^0, s^0) and μ^0 > 0 such that Ψ(x^0, s^0, μ^0) ⩽ τ;
begin
  x := x^0; s := s^0; μ := μ^0;
  while nμ ⩾ ε do
  begin
    μ := (1 − θ)μ;
    while Ψ(v) > τ do
    begin
      solve Newton system (NS) for Δx and Δs;
      determine a step size α;
      x := x + αΔx;
      s := s + αΔs;
    end
  end
end
```

Remark 2.7. One distinguishes IPMs as large-update methods when $\theta = \Theta(1)$ and small-update methods when $\theta = \Theta(1/\sqrt{n})$. The small-update methods have the best known iteration complexity, but in practice large-update methods are more efficient than small-update.

Remark 2.8. Up till recently, only algorithms based on the logarithmic barrier functions were considered, e.g., $\Psi(v) = \sum_{i=1}^{n}((v_i^2 - 1)/2 - \log v_i)$ [12]. In [10], self-regular barrier function was introduced for LO problems and the theory of self-regular was also extended to $P_*(\kappa)$ LCPs and they showed the best complexity for large-update IPMs for $P_*(\kappa)$ LCPs with some specific self-regular kernel function.

3. The kernel function and growth behavior

In this section we define a barrier function which is not a logarithmic barrier and not self-regular. We consider a univariate function $\psi(t) : D \to R_+$, with $R_{++} \subseteq D$ as follows,

$$\psi(t) = \frac{t^2 - 1}{2} + \frac{(e-1)^2}{e}\frac{1}{e^t - 1} - \frac{e-1}{e}, \quad t > 0.$$

To simplify the analysis we will restrict ourselves to the case where the proximity function $\Psi(v)$ is separable with identical coordinate functions. Thus, letting ψ denote the function on the coordinates, we have

$$\Psi(v) = \sum_{i=1}^{n} \psi(v_i).$$

We call the univariate function $\psi(t)$ the *kernel function* of the proximity function $\Psi(v)$. For $\psi(t)$ we have

$$\psi'(t) = t - \frac{(e-1)^2}{e}\frac{e^t}{(e^t - 1)^2},$$

$$\psi''(t) = 1 + \frac{(e-1)^2}{e}\frac{e^t(1 + e^t)}{(e^t - 1)^3},$$

$$\psi'''(t) = -\frac{(e-1)^2}{e}\frac{e^t(1 + 4e^t + e^{2t})}{(e^t - 1)^4}.$$

Since $\psi''(t) > 1$, $\psi(t)$ is strongly convex. Note that

$$\psi(1) = \psi'(1) = 0, \quad \psi'''(t) < 0.$$

And due to $\psi(1) = \psi'(1) = 0$, $\psi(t)$ is determined by the second derivative:

$$\psi(t) = \int_1^t \int_1^{\xi} \psi''(\varsigma)\, \mathrm{d}\varsigma\, \mathrm{d}\xi. \tag{3.1}$$

We define the norm-based proximity measure $\delta(v)$ as follows:

$$\delta(v) = \tfrac{1}{2}\|\nabla\Psi(v)\| = \tfrac{1}{2}\|\mathrm{d}x + \mathrm{d}s\|.$$

Note that since $\Psi(v)$ is strictly convex and minimal at $v = e$ we have

$$\Psi(v) = 0 \;\Leftrightarrow\; \delta(v) = 0 \;\Leftrightarrow\; v = e.$$

In the following lemma, we give a key property which is important in the analysis of the algorithm. The reader can refer to Lemma 2.2 in [1] for the proof.

Lemma 3.1. *Kernel function $\psi(t)$ is exponentially convex.*

Lemma 3.2 (*Bai et al. [1, Lemma 2.3]*). *The kernel function $\psi(t)$ as the following property*:

$$\psi(t) \leqslant \tfrac{1}{2}\psi'(t)^2, \quad t > 0.$$

Corollary 3.3 (*Bai et al. [1, Corollary 2.4]*). *We have*

$$\sqrt{\frac{\Psi(v)}{2}} \leqslant \delta(v).$$

Note that at the start of outer iteration of the algorithm, just before the update of μ with the factor $1-\theta$, we have $\Psi(v) \leqslant \tau$. Due to the update of μ the vector v is divided by the factor $\sqrt{1-\theta}$, with $0<\theta<1$, which in general leads to an increase in the value of $\Psi(v)$. Then, during the subsequent inner iterations, $\Psi(v)$ decreases until it passes the threshold τ again. Hence, during the course of the algorithm the largest values of $\Psi(v)$ occur just after the updates of μ. The following lemma give an estimate for the effect of a μ-update on the value of $\Psi(v)$.

Lemma 3.4 (*Bai et al. [1, Lemma 2.7]*). *Assume that* $0 \leqslant \theta < 1$ *and* $v_+ = v/(\sqrt{1-\theta})$. *Then we have*

$$\Psi(v_+) \leqslant \Psi(v) + \frac{\theta}{2(1-\theta)}(2\Psi(v) + 2\sqrt{2n\Psi(v)} + n).$$

We define

$$L(n,\theta,\tau) = \tau + \frac{\theta}{2(1-\theta)}(2\tau + 2\sqrt{2n\tau} + n).$$

Then by Lemma 3.4 and the assumption $\Psi(v) \leqslant \tau$ just before the update of μ, $\Psi(v_+) \leqslant L(n,\theta,\tau)$. Since $\tau = \mathrm{O}(n)$ and $0 \leqslant \theta < 1$, $L = \mathrm{O}(n)$.

4. Computation of the step size and the decrease

In this section we compute the feasible step size α such that the proximity function is decreasing and the bound for the decrease during inner iterations. Since $P_*(\kappa)$ LCPs are generalization of LO problems, we lose the orthogonality of vectors $\mathrm{d}x$ and $\mathrm{d}s$. So the analysis is different from LO case. After a damped step for fixed μ we have new iterates

$$x_+ = x + \alpha\Delta x, \quad s_+ = s + \alpha\Delta s.$$

From (2.1), we have

$$x_+ = x\left(e + \alpha\frac{\Delta x}{x}\right) = x\left(e + \alpha\frac{\mathrm{d}x}{v}\right) = \frac{x}{v}(v + \alpha\,\mathrm{d}x), \quad s_+ = s\left(e + \alpha\frac{\Delta s}{s}\right) = s\left(e + \alpha\frac{\mathrm{d}s}{v}\right) = \frac{s}{v}(v + \alpha\,\mathrm{d}s).$$

Then we get

$$v_+^2 = \frac{x_+ s_+}{\mu} = (v + \alpha\,\mathrm{d}x)(v + \alpha\,\mathrm{d}s).$$

Throughout the paper we assume that the step size α is such that the coordinates of the vectors $v + \alpha\,\mathrm{d}x$ and $v + \alpha\,\mathrm{d}s$ are positive. Hence by Lemma 3.1,

$$\Psi(v_+) = \Psi(\sqrt{(v + \alpha\,\mathrm{d}x)(v + \alpha\,\mathrm{d}s)}) \leqslant \tfrac{1}{2}(\Psi(v + \alpha\,\mathrm{d}x) + \Psi(v + \alpha\,\mathrm{d}s)).$$

For given $\mu > 0$ by letting $f(\alpha)$ be the difference of the new and old proximity measures, i.e.,

$$f(\alpha) = \Psi(v_+) - \Psi(v),$$

we have $f(\alpha) \leqslant f_1(\alpha)$, where

$$f_1(\alpha) := \tfrac{1}{2}(\Psi(v + \alpha\,\mathrm{d}x) + \Psi(v + \alpha\,\mathrm{d}s)) - \Psi(v).$$

Note that

$$f(0) = f_1(0) = 0.$$

For notational convenience, we denote by $\mathrm{d}x_i$ and $\mathrm{d}s_i$ ith components of vectors $\mathrm{d}x$ and $\mathrm{d}s$, respectively. By taking the derivative of $f_1(\alpha)$ with respect to α, we have

$$f_1'(\alpha) = \frac{1}{2}\sum_{i=1}^{n}(\psi'(v_i + \alpha\,\mathrm{d}x_i)\mathrm{d}x_i + \psi'(v_i + \alpha\,\mathrm{d}s_i)\mathrm{d}s_i).$$

From (2.3) and the definition of δ,

$$f_1'(0) = \tfrac{1}{2}\nabla\Psi(v)^{\mathrm{T}}(\mathrm{d}x + \mathrm{d}s) = -\tfrac{1}{2}\nabla\Psi(v)^{\mathrm{T}}\nabla\Psi(v) = -2\delta(v)^2. \tag{4.1}$$

By differentiating $f_1'(\alpha)$ with respect to α, we obtain

$$f_1''(\alpha) = \frac{1}{2}\sum_{i=1}^{n}(\psi''(v_i + \alpha\,\mathrm{d}x_i)\mathrm{d}x_i^2 + \psi''(v_i + \alpha\,\mathrm{d}s_i)\mathrm{d}s_i^2). \tag{4.2}$$

Since M is a $P_*(\kappa)$ matrix and $M\Delta x = \Delta s$ from (NS), for $\Delta x \in R^n$ we have

$$(1 + 4\kappa)\sum_{i\in J_+}\Delta x_i\Delta s_i + \sum_{i\in J_-}\Delta x_i\Delta s_i \geqslant 0,$$

where $J_+ = \{i \in J : \Delta x_i\Delta s_i \geqslant 0\}$, $J_- = J - J_+$. Since $\mathrm{d}x\,\mathrm{d}s = v^2\Delta x\Delta s/xs = \Delta x\Delta s/\mu$ and $\mu > 0$,

$$(1 + 4\kappa)\sum_{i\in J_+}\mathrm{d}x_i\,\mathrm{d}s_i + \sum_{i\in J_-}\mathrm{d}x_i\,\mathrm{d}s_i \geqslant 0. \tag{4.3}$$

For notational convenience we define

$$\delta := \delta(v), \quad \sigma_+ = \sum_{i\in J_+}\mathrm{d}x_i\,\mathrm{d}s_i, \quad \sigma_- = -\sum_{i\in J_-}\mathrm{d}x_i\,\mathrm{d}s_i.$$

In the following we compute the bound of $\|\mathrm{d}x\|$ and $\|\mathrm{d}s\|$. To compute this, we need the following technical lemma.

Lemma 4.1. *$\sigma_+ \leqslant \delta^2$ and $\sigma_- \leqslant (1 + 4\kappa)\delta^2$.*

Proof. By the definition of σ_+, σ_-, and δ,

$$\sigma_+ = \sum_{i\in J_+}\mathrm{d}x_i\,\mathrm{d}s_i \leqslant \frac{1}{4}\sum_{i\in J_+}(\mathrm{d}x_i + \mathrm{d}s_i)^2 \leqslant \frac{1}{4}\sum_{i=1}^{n}(\mathrm{d}x_i + \mathrm{d}s_i)^2 = \frac{1}{4}\|\mathrm{d}x + \mathrm{d}s\|^2 = \delta^2.$$

Since M is a $P_*(\kappa)$ matrix, from (4.3),

$$(1 + 4\kappa)\sigma_+ - \sigma_- \geqslant 0.$$

Thus

$$\sigma_- \leqslant (1 + 4\kappa)\sigma_+ \leqslant (1 + 4\kappa)\delta^2. \qquad \square$$

In the following lemma we compute the bound for $\|\mathrm{d}x\|$ and $\|\mathrm{d}s\|$.

Lemma 4.2. *$\sum_{i=1}^{n}(\mathrm{d}x_i^2 + \mathrm{d}s_i^2) \leqslant 4(1 + 2\kappa)\delta^2$, $\|\mathrm{d}x\| \leqslant 2\sqrt{1 + 2\kappa}\delta$, and $\|\mathrm{d}s\| \leqslant 2\sqrt{1 + 2\kappa}\delta$.*

Proof. Since $\delta(v) = \frac{1}{2}\|\mathrm{d}x + \mathrm{d}s\|$ and $\sum_{i\in J}\mathrm{d}x_i\,\mathrm{d}s_i = \sigma_+ - \sigma_-$,

$$2\delta = \|\mathrm{d}x + \mathrm{d}s\| = \sqrt{\sum_{i=1}^{n}(\mathrm{d}x_i + \mathrm{d}s_i)^2} = \sqrt{\sum_{i=1}^{n}(\mathrm{d}x_i^2 + \mathrm{d}s_i^2) + 2(\sigma_+ - \sigma_-)}.$$

From (4.3), $(1+4\kappa)\sigma_+ \geqslant \sigma_-$. Thus we have

$$2\delta \geqslant \sqrt{\sum_{i=1}^{n}(\mathrm{d}x_i^2+\mathrm{d}s_i^2)+2\left(\frac{1}{1+4\kappa}\sigma_- - \sigma_-\right)} = \sqrt{\sum_{i=1}^{n}(\mathrm{d}x_i^2+\mathrm{d}s_i^2)-\frac{8\kappa}{1+4\kappa}\sigma_-}.$$

If we square both sides, then we have

$$4\delta^2+\frac{8\kappa}{1+4\kappa}\sigma_- \geqslant \sum_{i=1}^{n}(\mathrm{d}x_i^2+\mathrm{d}s_i^2).$$

By Lemma 4.1,

$$4(1+2\kappa)\delta^2 \geqslant 4\delta^2+\frac{8\kappa}{1+4\kappa}\sigma_- \geqslant \sum_{i=1}^{n}(\mathrm{d}x_i^2+\mathrm{d}s_i^2).$$

So we have

$$2\sqrt{1+2\kappa}\delta \geqslant \sqrt{\sum_{i=1}^{n}(\mathrm{d}x_i^2+\mathrm{d}s_i^2)} \geqslant \|\mathrm{d}x\|,$$

and by the same way, we get $2\sqrt{1+2\kappa}\delta \geqslant \|\mathrm{d}s\|$. This completes the proof. □

To compute the upper bound for the difference of the new and old proximity measures, we need the following technical lemmas.

Lemma 4.3. $f_1''(\alpha) \leqslant 2(1+2\kappa)\delta^2\psi''(v_{\min}-2\alpha\sqrt{1+2\kappa}\delta)$.

Proof. From Lemma 4.2,

$$v_i+\alpha\,\mathrm{d}x_i \geqslant v_{\min}-2\alpha\sqrt{1+2\kappa}\delta \quad \text{and} \quad v_i+\alpha\,\mathrm{d}s_i \geqslant v_{\min}-2\alpha\sqrt{1+2\kappa}\delta.$$

By (4.2), $\psi'''(t)<0$, and Lemma 4.2,

$$\begin{aligned}
f_1''(\alpha) &= \frac{1}{2}\sum_{i=1}^{n}(\psi''(v_i+\alpha\,\mathrm{d}x_i)\,\mathrm{d}x_i^2+\psi''(v_i+\alpha\,\mathrm{d}s_i)\,\mathrm{d}s_i^2),\\
&\leqslant \frac{1}{2}\sum_{i=1}^{n}(\psi''(v_{\min}-2\alpha\sqrt{1+2\kappa}\delta)\,\mathrm{d}x_i^2+\psi''(v_{\min}-2\alpha\sqrt{1+2\kappa}\delta)\,\mathrm{d}s_i^2),\\
&= \frac{1}{2}\psi''(v_{\min}-2\alpha\sqrt{1+2\kappa}\delta)\sum_{i=1}^{n}(\mathrm{d}x_i^2+\mathrm{d}s_i^2),\\
&\leqslant \frac{1}{2}\psi''(v_{\min}-2\alpha\sqrt{1+2\kappa}\delta)4(1+2\kappa)\delta^2,\\
&= 2(1+2\kappa)\delta^2\psi''(v_{\min}-2\alpha\sqrt{1+2\kappa}\delta). \qquad \square
\end{aligned}$$

Lemma 4.4. $f_1'(\alpha) \leqslant 0$ *if* α *is satisfying*

$$-\psi'(v_{\min}-2\alpha\delta\sqrt{1+2\kappa})+\psi'(v_{\min}) \leqslant \frac{2\delta}{\sqrt{1+2\kappa}}. \tag{4.4}$$

Proof. Using (4.1), Lemma 4.3, and the assumption,

$$\begin{aligned} f_1'(\alpha) &= f_1'(0) + \int_0^\alpha f_1''(\xi)\,\mathrm{d}\xi, \\ &\leqslant -2\delta^2 + 2(1+2\kappa)\delta^2 \int_0^\alpha \psi''(v_{\min} - 2\xi\sqrt{1+2\kappa}\delta)\,\mathrm{d}\xi, \\ &= -2\delta^2 - \sqrt{1+2\kappa}\delta \int_0^\alpha \psi''(v_{\min} - 2\xi\sqrt{1+2\kappa}\delta)\,\mathrm{d}(v_{\min} - 2\xi\sqrt{1+2\kappa}\delta), \\ &= -2\delta^2 - \sqrt{1+2\kappa}\delta(\psi'(v_{\min} - 2\alpha\delta\sqrt{1+2\kappa}) - \psi'(v_{\min})), \\ &\leqslant -2\delta^2 + \sqrt{1+2\kappa}\delta\frac{2\delta}{\sqrt{1+2\kappa}}, \\ &= 0. \end{aligned}$$

We get the desired result. □

In the following lemma, we compute the feasible step size α such that the proximity measure is decreasing when we take a new iterate for fixed μ.

Lemma 4.5. *Let* $\rho : [0, \infty) \to (0, 1]$ *denote the inverse function of the restriction of* $-\frac{1}{2}\psi'(t)$ *to the interval* $(0, 1]$. *Then the largest step size* α *that satisfies* (4.4) *is given by*

$$\bar{\alpha} := \frac{1}{2\delta\sqrt{1+2\kappa}}\left(\rho(\delta) - \rho\left(\left(1 + \frac{1}{\sqrt{1+2\kappa}}\right)\delta\right)\right). \tag{4.5}$$

Proof. We want to compute the step size α such that (4.4) holds with α as large as possible. The derivative of the left hand side in (4.4) with respect to α is

$$2\delta\sqrt{1+2\kappa}\psi''(v_{\min} - 2\alpha\delta\sqrt{1+2\kappa}) > 0$$

since $\psi'' > 0$. Hence the left hand side in (4.4) is monotone increasing in α. So the largest possible value of α satisfying (4.4) occurs when

$$-\psi'(v_{\min} - 2\alpha\delta\sqrt{1+2\kappa}) + \psi'(v_{\min}) = \frac{2\delta}{\sqrt{1+2\kappa}}. \tag{4.6}$$

The derivative of the left hand side in (4.6) with respect to $v_{\min}$ is

$$-\psi''(v_{\min} - 2\alpha\delta\sqrt{1+2\kappa}) + \psi''(v_{\min}) < 0.$$

Since $\psi''' < 0$, the left hand side in (4.6) is decreasing in $v_{\min}$. This implies that with δ fixed if $v_{\min}$ gets smaller, then α gets smaller. Note that by the definition of δ and $\Psi(v)$,

$$\delta = \frac{1}{2}\|\nabla\Psi(v)\| = \frac{1}{2}\sqrt{\sum_{i=1}^n (\psi'(v_i))^2} \geqslant \frac{1}{2} \mid \psi'(v_{\min}) \mid \geqslant -\frac{1}{2}\psi'(v_{\min}).$$

Equality holds if and only if $v_{\min}$ is the only coordinate in v which is different from 1 and $v_{\min} \leqslant 1$, i.e., $\psi'(v_{\min}) \leqslant 0$. Hence when $v_{\min}$ satisfies

$$-\tfrac{1}{2}\psi'(v_{\min}) = \delta, \tag{4.7}$$

the smallest step size α occurs. In this case by (4.7) and the definition of ρ,

$$v_{\min} = \rho(\delta). \tag{4.8}$$

From (4.6) and (4.7),

$$-\frac{1}{2}\psi'(v_{\min} - 2\alpha\delta\sqrt{1+2\kappa}) = \delta\left(1 + \frac{1}{\sqrt{1+2\kappa}}\right). \tag{4.9}$$

Then by (4.9) and the definition of ρ,

$$v_{\min} - 2\alpha\delta\sqrt{1+2\kappa} = \rho\left(\left(1 + \frac{1}{\sqrt{1+2\kappa}}\right)\delta\right).$$

Thus by (4.8), the largest step size α is given as follows:

$$\alpha = \frac{1}{2\delta\sqrt{1+2\kappa}}\left(\rho(\delta) - \rho\left(\left(1 + \frac{1}{\sqrt{1+2\kappa}}\right)\delta\right)\right). \qquad \square$$

In the following lemma we compute the lower bound for $\bar{\alpha}$ in Lemma 4.5. For notational convenience we denote $\gamma := (e-1)^2/e$.

Lemma 4.6. *Let ρ and $\bar{\alpha}$ be as defined in Lemma* 4.5. *Then for $a = 1 + 1/\sqrt{1+4\kappa}$ we have*

$$\bar{\alpha} \geqslant \frac{1}{1+2\kappa}\frac{1}{1 + (2.3)\gamma^{-1/2}(2a\delta+1)^{3/2}}.$$

Proof. By the definition of ρ,

$$-\psi'(\rho(\delta)) = 2\delta.$$

Taking the derivative with respect to δ, we get

$$-\psi''(\rho(\delta))\rho'(\delta) = 2.$$

Since $\psi'' > 0$, we have

$$\rho'(\delta) = -\frac{2}{\psi''(\rho(\delta))} < 0. \tag{4.10}$$

Hence ρ is monotonically decreasing in δ. By (4.5), the fundamental theorem of calculus, and (4.10), we have

$$\begin{aligned}
\bar{\alpha} &= \frac{1}{2\delta\sqrt{1+2\kappa}}\left(\rho(\delta) - \rho\left(\left(1 + \frac{1}{\sqrt{1+2\kappa}}\right)\delta\right)\right) \\
&= \frac{1}{2\delta\sqrt{1+2\kappa}}\int_{(1+1/\sqrt{1+2\kappa})\delta}^{\delta} \rho'(\xi)\,\mathrm{d}\xi \\
&= \frac{1}{\delta\sqrt{1+2\kappa}}\int_{\delta}^{(1+1/\sqrt{1+2\kappa})\delta} \frac{\mathrm{d}\xi}{\psi''(\rho(\xi))}.
\end{aligned}$$

To obtain a lower bound for $\bar{\alpha}$, we want to replace the argument of the last integral by its minimal value. Since $\delta \leqslant \xi \leqslant (1 + 1/\sqrt{1+2\kappa})\delta$ and ρ is monotonically decreasing in δ,

$$\rho(\xi) \geqslant \rho\left(\left(1 + \frac{1}{\sqrt{1+2\kappa}}\right)\delta\right).$$

Since $\psi''' < 0$, i.e., ψ'' is monotonically decreasing,

$$\psi''(\rho(\xi)) \leqslant \psi''\left(\rho\left(\left(1 + \frac{1}{\sqrt{1+2\kappa}}\right)\delta\right)\right).$$

Hence, we have

$$\frac{1}{\psi''(\rho(\xi))} \geqslant \frac{1}{\psi''(\rho((1+1/\sqrt{1+2\kappa})\delta))}.$$

Therefore, we have

$$\begin{aligned}\bar{\alpha} &= \frac{1}{\delta\sqrt{1+2\kappa}} \int_{\delta}^{(1+1/\sqrt{1+2\kappa})\delta} \frac{\mathrm{d}\xi}{\psi''(\rho(\xi))} \\ &\geqslant \frac{1}{\delta\sqrt{1+2\kappa}} \frac{1}{\psi''(\rho((1+1/\sqrt{1+2\kappa})\delta))} \int_{\delta}^{(1+1/\sqrt{1+2\kappa})\delta} \mathrm{d}\xi \\ &= \frac{1}{1+2\kappa} \frac{1}{\psi''(\rho((1+1/\sqrt{1+2\kappa})\delta))}. \end{aligned} \tag{4.11}$$

Let $\rho(a\delta) = t$. Then by the definition of ρ, $a\delta = -\frac{1}{2}\psi'(t)$ and $0 < t \leqslant 1$. By the definition of $\psi'(t)$ and γ, $2a\delta = -t + \gamma e^t/(e^t-1)^2$. Then we have

$$\frac{e^t}{(e^t-1)^2} = \gamma^{-1}(2a\delta + t). \tag{4.12}$$

By (4.11), the definition of $\psi''(t)$, (4.12), and $e^{t/2} + e^{-t/2} \leqslant 2.3$ for $0 < t \leqslant 1$,

$$\begin{aligned}\bar{\alpha} &\geqslant \frac{1}{1+2\kappa}\frac{1}{\psi''(t)} = \frac{1}{1+2\kappa}\frac{1}{1+\gamma e^t(1+e^t)/(e^t-1)^3} = \frac{1}{1+2\kappa}\frac{1}{1+\gamma e^{3t/2}(e^{t/2}+e^{-t/2})/(e^t-1)^3} \\ &= \frac{1}{1+2\kappa}\frac{1}{1+\gamma(e^{t/2}+e^{-t/2})(e^t/(e^t-1)^2)^{3/2}} = \frac{1}{1+2\kappa}\frac{1}{1+\gamma(e^{t/2}+e^{-t/2})\gamma^{-3/2}(2a\delta+t)^{3/2}} \\ &\geqslant \frac{1}{1+2\kappa}\frac{1}{1+(2.3)\gamma^{-1/2}(2a\delta+t)^{3/2}} \geqslant \frac{1}{1+2\kappa}\frac{1}{1+(2.3)\gamma^{-1/2}(2a\delta+1)^{3/2}}. \end{aligned}$$

The second inequality follows from the fact that $0 < t \leqslant 1$. □

Define

$$\tilde{\alpha} = \frac{1}{1+2\kappa}\frac{1}{1+(2.3)\gamma^{-1/2}(2a\delta+1)^{3/2}}. \tag{4.13}$$

Then by Lemma 4.6, we have $\bar{\alpha} \geqslant \tilde{\alpha}$. We will use $\tilde{\alpha}$ as the default step size in the Algorithm. To evaluate the decrease of the proximity function value, we cite the following result in [10].

Lemma 4.7 (*Peng et al. [10, Lemma 1.3.3]*). *Let $h(t)$ be a twice differentiable convex function with $h(0) = 0$, $h'(0) < 0$ and $h(t)$ attain its global minimum at $t^* > 0$. If $h''(t)$ is increasing for $t \in [0, t^*]$, then*

$$h(t) \leqslant \frac{t h'(0)}{2}, \quad 0 \leqslant t \leqslant t^*.$$

Lemma 4.8. *If the step size α is such that $\alpha \leqslant \bar{\alpha}$, then*

$$f(\alpha) \leqslant -\alpha\delta^2.$$

Proof. Define the univariate function h as follows:

$$h(0) = f_1(0) = 0, \quad h'(0) = f_1'(0) = -2\delta^2, \quad h''(\alpha) = 2(1+2\kappa)\delta^2\psi''(v_{\min} - 2\alpha\sqrt{1+2\kappa}\delta).$$

By Lemma 4.3 , $f_1''(\alpha) \leqslant h''(\alpha)$. So we have $f_1'(\alpha) \leqslant h'(\alpha)$ and $f_1(\alpha) \leqslant h(\alpha)$. By the definition of $h(\alpha)$ and $\psi''(t) > 1$, $h''(\alpha) \geqslant 2(1+\kappa)\delta^2$. This implies that $h(\alpha)$ is strongly convex and hence $h(\alpha)$ attains its global minimum for some

$\alpha^* > 0$. Taking $\alpha \leqslant \bar{\alpha}$, with $\bar{\alpha}$ as defined in Lemma 4.5, using the fundamental theorem of calculus, and Lemma 4.4, we have

$$\begin{aligned} h'(\alpha) &= h'(0) + \int_0^\alpha h''(\xi)\,\mathrm{d}\xi \\ &= -2\delta^2 + 2(1+2\kappa)\delta^2 \int_0^\alpha \psi''(v_{\min} - 2\xi\sqrt{1+2\kappa}\delta)\,\mathrm{d}\xi \\ &= -2\delta^2 - \sqrt{1+2\kappa}\delta(\psi'(v_{\min} - 2\alpha\sqrt{1+2\kappa}\delta) - \psi'(v_{\min})) \\ &\leqslant -2\delta^2 + \sqrt{1+2\kappa}\delta\frac{2\delta}{\sqrt{1+2\kappa}} \\ &= 0. \end{aligned}$$

Since $h'''(\alpha) = -4(1+2\kappa)^{3/2}\delta^3\psi'''(v_{\min} - 2\alpha\sqrt{1+2\kappa}\delta)$ and $\psi''' < 0$, $h''(\alpha)$ is increasing in α. Hence by Lemma 4.7, we may write

$$f_1(\alpha) \leqslant h(\alpha) \leqslant \tfrac{1}{2}\alpha h'(0) = -\alpha\delta^2.$$

Since $f(\alpha) \leqslant f_1(\alpha)$, the proof is completed. □

In the following theorem we have the upper bound for the difference $f(\alpha)$ between the new and old proximity measures during an inner iteration by using Lemma 4.8 and (4.13).

Theorem 4.9. *Let $\tilde{\alpha}$ be a step size as defined in* (4.13). *Then we have*

$$f(\tilde{\alpha}) \leqslant -\frac{1}{1+2\kappa}\frac{\delta^2}{1+(2.3)\gamma^{-1/2}(2a\delta+1)^{3/2}}. \tag{4.14}$$

By simple calculation we can show the following property.

Lemma 4.10. *The right-hand side in* (4.14) *is monotonically decreasing in δ.*

Proof. Define

$$g(\delta) = \frac{\delta^2}{1+(2.3)\gamma^{-1/2}(2a\delta+1)^{3/2}}.$$

Then

$$g'(\delta) = \frac{2\delta + \delta\gamma^{-1/2}(2a\delta+1)^{1/2}(2.3a\delta+1)}{\{1+(2.3)\gamma^{-1/2}(2a\delta+1)^{3/2}\}^2} > 0.$$

This implies that $g(\delta)$ is monotonically increasing and hence the right hand side in (4.14) is monotonically decreasing. □

5. Complexity analysis

In this section we analyze the complexity of the algorithm. We cite the following lemma in [10] to obtain iteration bounds for the algorithm.

Lemma 5.1 (*Peng et al. [10, Proposition 1.3.2]*). *Let $t_0, t_1, \ldots, t_K$ be a sequence of positive numbers such that*

$$t_{k+1} \leqslant t_k - \beta t_k^{1-\tilde{\gamma}}, \quad k = 0, 1, \ldots, K-1,$$

where $\beta > 0$ and $0 < \tilde{\gamma} \leqslant 1$. Then $K \leqslant \lfloor t_0^{\tilde{\gamma}}/\beta\tilde{\gamma} \rfloor$.

We define the value of $\Psi(v)$ after the μ-update as Ψ_0 and the subsequent values in the same outer iteration are denoted as Ψ_k, $k = 1, 2, \ldots$. Let K denote the total number of inner iterations in the outer iteration. Then we have

$$\Psi_{K-1} > \tau, \quad 0 \leqslant \Psi_K \leqslant \tau.$$

In the following lemma, we compute the upper bound for the total number of inner iterations which we needed to return to the τ-neighborhood, i.e., $\Psi(v) \leqslant \tau$ after a μ-update.

Lemma 5.2. *Let K be the total number of inner iterations in an outer iteration. Then we have*

$$K \leqslant 88(1 + 2\kappa)\Psi_0^{3/4},$$

where Ψ_0 denotes the value of $\Psi(v)$ after the μ-update.

Proof. By Theorem 4.9, Lemma 4.10, and Corollary 3.3 with $\Psi := \Psi(v)$,

$$f(\tilde{\alpha}) \leqslant -\frac{1}{1+2\kappa}\frac{(\sqrt{\Psi/2})^2}{1+(2.3)\gamma^{-1/2}(2a\sqrt{\Psi/2}+1)^{3/2}}. \tag{5.1}$$

By assuming $\Psi_0 \geqslant \Psi \geqslant \tau \geqslant 1$ with $a = 1 + 1/\sqrt{1+2\kappa} \geqslant 1$,

$$2a\sqrt{\frac{\Psi}{2}} + 1 \leqslant \sqrt{2\Psi}a + \sqrt{\Psi} \leqslant 2\sqrt{2\Psi}a. \tag{5.2}$$

By (5.1) , (5.2), $\Psi \geqslant 1$, and the definition of γ,

$$\begin{aligned} f(\tilde{\alpha}) &\leqslant -\frac{1}{2(1+2\kappa)}\frac{\Psi}{1+(2.3)\gamma^{-1/2}(2\sqrt{2}a\sqrt{\Psi})^{3/2}} = -\frac{1}{2(1+2\kappa)}\frac{\Psi}{1+(2.3)\gamma^{-1/2}2^{9/4}a^{3/2}\Psi^{3/4}} \\ &\leqslant -\frac{1}{2(1+2\kappa)}\frac{\Psi}{\Psi^{3/4}+(2.3)\gamma^{-1/2}2^{9/4}a^{3/2}\Psi^{3/4}} \leqslant -\frac{1}{2(1+2\kappa)(1+11a^{3/2})}\Psi^{1/4}. \end{aligned}$$

Thus it follows that

$$\Psi_{k+1} \leqslant \Psi_k - \beta\Psi_k^{1-\tilde{\gamma}}, \quad k = 0, 1, 2, \ldots, K-1,$$

with $\beta = 1/2(1+2\kappa)(1+11a^{3/2})$ and $\tilde{\gamma} = \frac{3}{4}$. Hence by Lemma 5.1 and $a \leqslant 2$, the total number K of inner iterations is bounded above by

$$K \leqslant \tfrac{8}{3}(1+2\kappa)(1+11a^{3/2})\Psi_0^{3/4} \leqslant 88(1+2\kappa)\Psi_0^{3/4}.$$

This completes the proof. □

The upper bound for the total number of iterations is obtained by multiplying the number K by the number of central path parameter updates. If the central path parameter μ has the initial value μ^0 and is updated by multiplying $1-\theta$, with $0 < \theta < 1$, then after at most

$$\left\lceil \frac{1}{\theta}\log\frac{n\mu^0}{\varepsilon} \right\rceil$$

iterations we have $n\mu \leqslant \varepsilon$. Thus the total number of iterations is bounded above by

$$\frac{K}{\theta}\log\frac{n\mu^0}{\varepsilon} \leqslant \frac{88}{\theta}(1+2\kappa)\Psi_0^{3/4}\log\frac{n\mu^0}{\varepsilon}.$$

By Lemma 3.4, $\Psi_0 \leqslant L(n, \theta, \tau)$. In the following we have main result.

Theorem 5.3. *Let a linear complementarity problem for any $P_*(\kappa)$ matrix M be given, where $\kappa \geqslant 0$. Assume that a strictly feasible starting point (x^0, s^0) is available with $\Psi(x^0, s^0, \mu^0) \leqslant \tau$ for some $\mu^0 > 0$. Then the total number of iterations to obtain a feasible solution such that $n\mu \leqslant \varepsilon$ for our Algorithm is bounded above by*

$$\left\lceil 88(1+2\kappa)\left(\tau + \frac{\theta}{2(1-\theta)}(2\tau + 2\sqrt{2n\tau} + n)\right)^{3/4}\right\rceil \left\lceil \frac{1}{\theta}\log\frac{n\mu^0}{\varepsilon}\right\rceil.$$

Remark 5.4. For large-update methods (when $\tau = \mathrm{O}(n)$ and $\theta = \Theta(1)$) we get the polynomial complexity $\mathrm{O}((1+2\kappa)n^{3/4}\log(n/\varepsilon))$ which is the similar complexity for LO.

6. Concluding remarks

In this paper we extended the theory of IPMs based on a new proximity function in [1] to $P_*(\kappa)$ LCPs. New search directions and proximity measures are proposed based on the kernel function which is not logarithmic barrier nor self-regular. Our results show that if a strictly feasible starting point is available, then our Algorithm can identify the ε-approximate solution with the similar polynomial complexity bound for large-update method for the LO case and improve the results of the classical primal-dual algorithm based on logarithmic barrier function. The same approach presented in this paper can be applied for the recent kernel functions presented in [2]. By using these kernel functions we obtain the same complexity as obtained for LO problems.

Further research will be on the extension to more general classes of problems and the numerical implementation of the algorithm.

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